

Coherence resonance in a unijunction transistor relaxation oscillator

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The phenomenon of coherence resonance is investigated in an unijunction transistor relaxation oscillator and quantified by estimating the normal variance (NV). Depending on the measuring points, two types of NV curves have been obtained. We have observed that the degradations in coherency at higher noise amplitudes in our system is probably the result of direct interference of coherent oscillations and the stochastic perturbation. Degradation of coherency may be minimal if this direct interference of noise and coherent oscillations is eliminated.

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The study of the constructive role of noise, which is generally termed as coherence resonance, in threshold or excitable systems has received considerable attention in the last 20 years and has been observed in many physical, chemical, and biological and electronics systems [1–20]. The basic characteristic of a threshold system is that it shows fixed point and limit cycle attractors below and above a threshold, respectively. When the system resides at a fixed point, it reacts to external perturbations in two different ways depending on the perturbation amplitude. When the amplitude is small to cross the threshold, the system remains at a stable state; but if it is large enough to cross, a large amplitude excursion is produced in the output through limit cycle oscillations, before settling back to the fixed point. Application of the time-dependent external stochastic perturbation produces coherent limit cycle oscillations, depending on the amplitude of the perturbation, and maximum coherency has been observed for an optimum noise and coherency decreases for higher noise amplitudes [5,14–21]. The decrease in the coherency with an increase in noise intensity was explained as due to the increase in variation of the excursion time of the limit cycles for small threshold and large excursion time where an increase in mean excursion time is negligible [5]. But if stochastic perturbation helps the system only to cross the threshold, then, at higher amplitudes, the system will remain at an excitable state [22] producing only limit cycle oscillations, which has maximum coherency that contradicts the above explanation.

In this Rapid Communication we have presented experimental results that is consistent with latter explanation by means of a unijunction transistor relaxation oscillator (UJT-RO), which exhibits relaxation oscillations over a certain range of emitter voltages and is used extensively in triggering circuits [23,24]. We also show here that the destruction of coherency is due to direct interference of noise on the limit cycle oscillations, which is consistent with experimental results presented in Ref. [18].

The experiment has been carried out in an UJT-RO whose schematic circuit diagram has been shown in Fig. 1. For the present experiments, the oscillator was operated at $V_{BB} = 4.05$ V. Depending on the voltage (V_E) between emitter (E) and base 1 (B1), periodic oscillations were observed and the outputs were recorded at two points E (V_{EO}) and B1 (V_{B1O}), respectively. V_E which is the control parameter in the present experiment, can be varied by using a variable resistance R_T . A noise source was coupled at point E through the capacitor C_1 for noise-induced experiments.

Before studying the noise-invoked resonance dynamics, we characterized the autonomous behavior of the oscillator. Depending on the control parameter (V_E), the relaxation oscillations were observed for a particular range of V_E . Figure 2 (left panel) shows the experimental bifurcation diagram using V_E as the control parameter and is seen that, around $V_E \approx 2.5$ V, limit cycle oscillations were observed and vanish at about $V_E \approx 3$ V. The time period of these oscillations increases with V_E . The graph of $\ln T$ vs $\ln V_E$ (right panel) follows a power-law behavior with an exponent of ≈ 0.99 . So $V_E \approx 3$ V defines the threshold of the system for the particular choice of parameters.

To summarize, at V_E values below 2.5 V and above 3 V, excitable fixed point response was found. For the purpose of

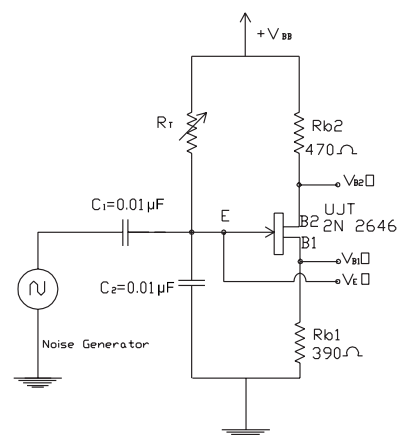


FIG. 1. Circuit description of the UJT relaxation oscillator.

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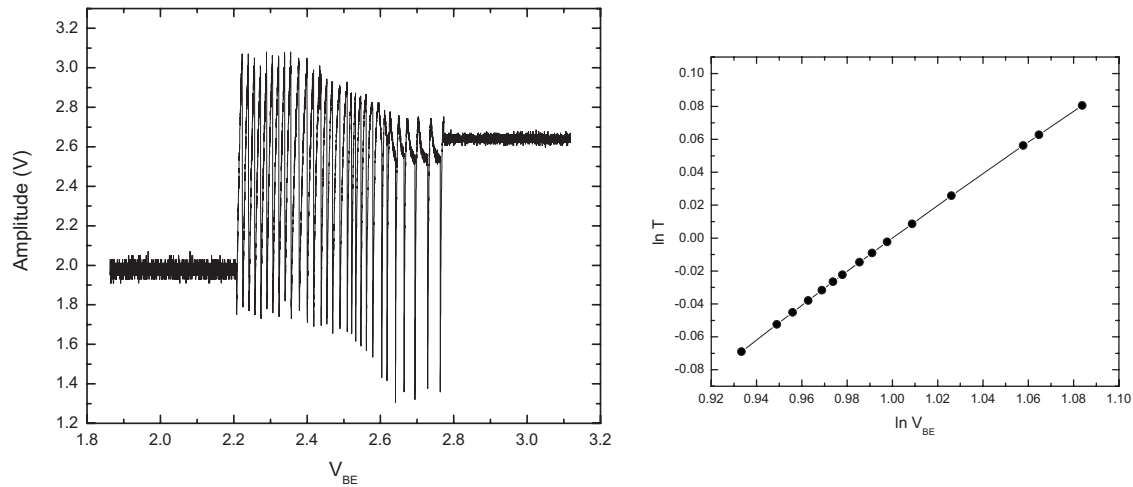


FIG. 2. Left panel: experimental bifurcation diagram constructed by increasing the voltage V_E at the output terminal $V_{E}O$. It shows that the parameters are divided into two regions: the fixed point region where there are no oscillations in the output and the oscillatory region where oscillations are observed. Right panel: $\ln T$ vs $\ln V_E$ curve fitted by a straight line indicates a power-law behavior of the increment of time period (T).

the present experiments, the set point was kept above the threshold ($V_E \approx 3$ V) and noise induced coherent responses were recorded at $V_{E}O$ and $V_{B1}O$, respectively.

For the experiments on coherence resonance, the resistance R_T was chosen such that the output of the UJT-RO exhibits a fixed point behavior in the absence of noise and a set point was chosen far from the threshold, so that the system always remains in a stable state under the influence of the intrinsic noise and parametric drifts. A Gaussian noise was superimposed on the bias voltage through the capacitor C_1 [Fig. 1] and the regularity of the provoked dynamics, which depends on noise intensity, was analyzed. The normalized variance (NV) was used to quantify the induced coherence. It is defined as $NV = \text{std}(t_p) / \text{mean}(t_p)$, where, t_p is the time taken between successive peaks. It is evident that the more the coherent the induced oscillations, the lower the value of the computed NV [14]. For pure oscillations, NV tends to zero.

Figures 3(a)–3(c) (left panel) show the time series of the output recorded at $V_{E}O$ for different noise levels and Fig. 3(d) is the experimental NV curve as a function of noise amplitude. Point (a) in Fig. 3(d) corresponds to the time series shown in Fig. 3(a) and is associated with a low level of noise where the activation threshold is seldom crossed, generating a sparsely populated irregular spike sequence. As the noise amplitude is increased, the NV decreases and reaches a minimum. Figure 3(b) shows the time series that corresponds to point (b) in Fig. 3(d) for maximum regularity at an optimum noise level. At higher amplitudes of the superimposed noise, the observed regularity is destroyed as manifested by an increase in the NV; labeled as (c) in Fig. 3(d), and the corresponding time series has been shown in Fig. 3(c). This is a consequence of the dynamics being dominated by noise.

Figure 3(d) shows that the system attains a coherent state with an increase in noise amplitude and stays at that state for a wide range of amplitudes before being degraded at higher

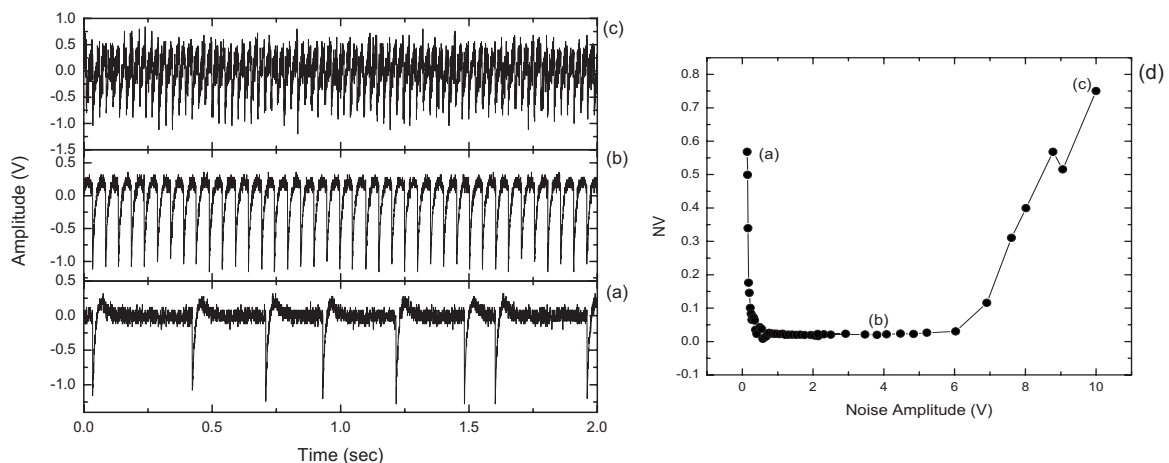


FIG. 3. Emergence of coherence resonance for the output recorded at emitter ($V_{E}O$): the right panel shows the NV as a function of noise amplitude for the experiments performed at $V_E = 3.142$ V. Left panel: the time series of the output for (a) low- (0.14 V), (b) optimum- (3.81 V), and (c) high-level (10 V) noise.

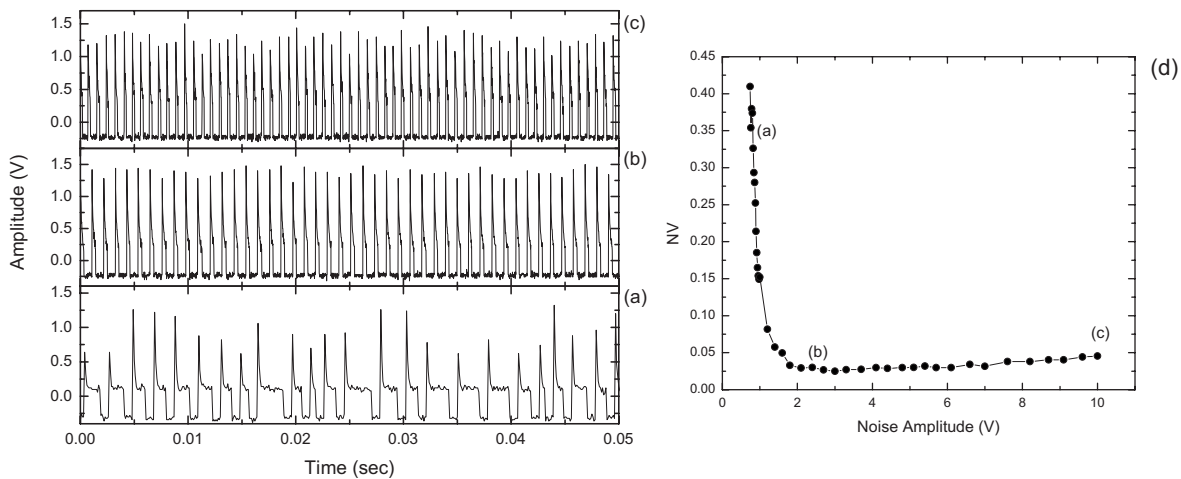


FIG. 4. Emergence of coherence resonance for the output recorded at base 1 ($V_{B1}O$): the right panel shows the NV as a function of noise amplitude for the experiments performed at $V_E=3.142$ V. Left panel: the time series of the output for (a) low- (0.76 V), (b) optimum- (2.4 V), and (c) high-level (10 V) noise.

values. Initially, the interpeak time is dominated by the activation time of the limit cycle oscillations, which varies significantly at low noise levels, and with an increase in the noise amplitudes the activation time decreases resulting in a rapid fall of NV [5,20]. When the noise amplitude is such that it crosses the threshold very often, i.e., number of crossing is much larger than the excursion frequency of the oscillations, it remains in the excited state, resulting in coherent oscillations leading to a flat minimum in the NV curve [Fig. 3(d)]. Direct interference of the strong noise distorts the limit cycle oscillations, particularly the peaks, and since the excursion time is almost independent of noise amplitude [5,20] the increase in NV at higher noise is probably due to the increase in the variation of the interpeak distances of the distorted peaks of the limit cycle oscillations [18].

Figures 4(a)–4(c) show the time series of the output recorded at $V_{B1}O$ of the oscillator for different noise levels and Fig. 4(d) is the experimental NV curve as a function of noise amplitude. As noise is increased from its minimum value, the NV rapidly reaches a minimum and remains constant and

shows a slight tendency to increase at higher noise amplitudes. Figure 4(a) shows the time series of irregular spiking at low noise levels and its corresponding NV point is (a) in Fig. 4(d). Figures 4(b) and 4(c) show the time series for two NV points (b) and (c) in Fig. 4(d) at moderate and maximum noises, respectively. It is seen that increasing the noise amplitude does not result in a significant difference in the measure of the coherency, i.e., in NV. Figure 4(c) shows that there is no significant distortion in the limit cycle at higher intensity of noise, although it is prominent in the first case [Fig. 3(c)], and this is probably because noise is blocked by some internal self-organization of the UJT, which prevents the destructive effect of the stochastic perturbation on the oscillations. In this case the higher amplitude noise only invokes permanent excitation in the system, resulting in coherent oscillations, and hence small NV. This also shows the robustness of the UJT against the destructive effect of noise in triggering circuits.

These observations indicate that the destructive effect of large stochastic perturbation probably depends on the system

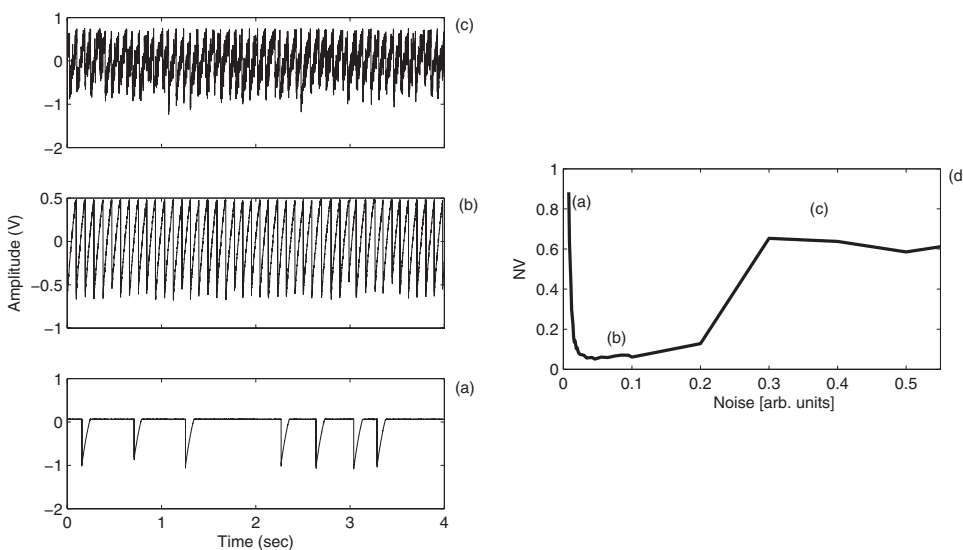


FIG. 5. Emergence of coherence resonance in the PSPICE simulation for the output recorded (V_E): the right panel shows the NV as a function of noise amplitude for the experiments performed at $V_E=2.75$ V. Left panel: the time series of the output for (a) low-, (b) optimum-, and (c) high-level noise.

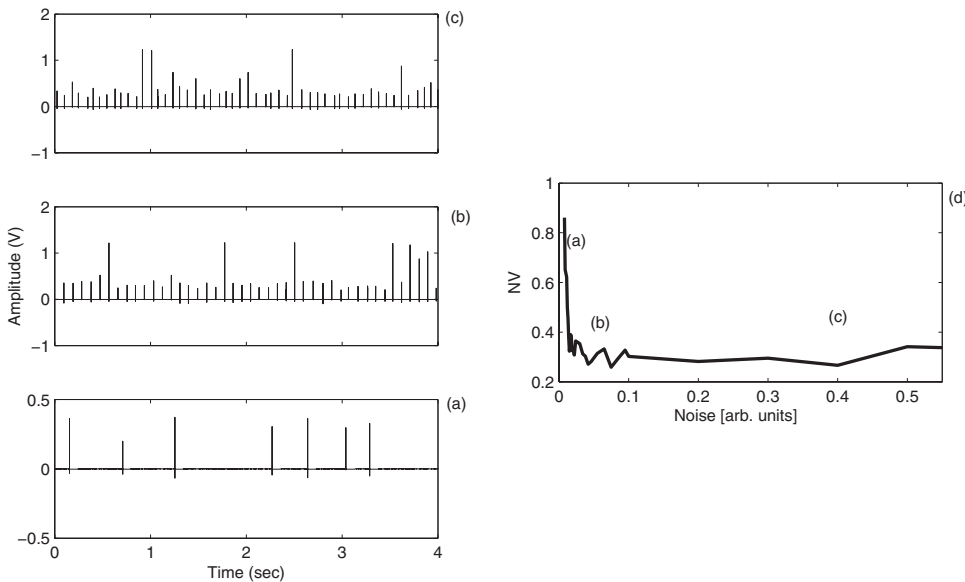


FIG. 6. Emergence of coherence resonance for the output recorded at base 1 ($V_{B1}O$): the right panel shows the NV as a function of noise amplitude for the experiments performed at $V_E=2.75$ V. Left panel: The time series of the output for (a) low-, (b) optimum-, and (c) high-level noise.

properties, relevant parameters, etc., and the system that can block their direct interference may not exhibit a significant increase in NV at higher amplitudes.

To validate our experimental results, PSPICE simulation of the UJT-RO was carried out using the same values for the capacitors and the resistors as in the real experiments. The only exception in the simulation was that R_T was chosen slightly higher than the experimental value to obtain relaxation oscillations. The relaxation oscillations were observed around $V_E \approx 2.6$ V and ceased around ≈ 2.65 V. For noise-induced experiments, V_E was set at 2.75 V and noise generated using MATLAB was fed into the circuit and measurement was performed at two points E and B1, respectively. Figure 5(d) shows the PSPICE simulated NV curve and Figs. 5(a)–5(c) show the three time series corresponding to points (a)–(c) shown on the same NV plot [Fig. 5(d)] for the data recorded at V_E . The NV curve and the time series for the simulated results show almost similar features of the experimental measurements shown in Fig. 3. Similarly, Fig. 6(d) shows the simulated NV curve for the data recorded at V_{B1}

and Figs. 6(a)–6(c) show the three time series corresponding to points (a)–(c) shown on the same NV plot [Fig. 6(d)]. Both PSPICE simulated NV curves [Figs. 5(d) and 6(d)] show a behavior very similar to the experimental NV curves [Figs. 3(d) and 4(d)].

In conclusion, the effect of noise has been studied experimentally in UJT-RO, demonstrating the emergence of CR via purely stochastic fluctuations. PSPICE simulation also shows similar kind of behavior. We have also shown that, depending on system properties, coherence may or may not be destroyed at high amplitude noise. Both kinds of NV may be observed in the same system depending on the relevant parameters.

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